Forty Ferrero Rocher, four kilograms of macaroni and the Tower of Hanoi - some thoughts on developing active Numeracy teaching activities.

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The title of this piece stems from being en route to a recent session with some Numeracy teachers on an In-Service course where we were looking at ways of teaching algebra. My teaching resources were all packed into a useful trolley which was rather heavy that evening and I realised that I was carrying the items listed in the title, which was why it weighed so much. The reasons for using these particular resources will be explained a little later.

My personal interest in active forms of numeracy teaching pre-dates my current role as a teacher educator and stems from two influences in the late 1990s. What I have been trying to research and develop over the past few years is in response to difficulties that I faced in my previous role as a Numeracy tutor in the Adult and Community section of Huddersfield Technical College in trying to teach many concepts in Numeracy such as multiplication, fraction operations and the understanding of shape and space problems.

Many of my students were becoming 'stuck' in terms of their progression at the stage where they had to move from four rules of number work to the complex application of those skills to higher order tasks. In terms of Bloom's taxonomy (1956) they had mastered the knowledge about numbers but were not effectively moving on to the comprehension, in that they could not easily transfer that knowledge to a new context and thus could not go on to the applying, analysing and synthesising needed to cope with the higher levels of assessment.

Mathematics teaching and in particular the difficulties that many people have with maths has long been a subject of study by members of the teaching profession and educational psychologists. Maths is a subject that invokes emotions like no other in education as (Ahmed 1987, p.22) succinctly states,

> Maths is a subject closely related to failure, and is also socially acceptable to be bad at mathematics. I think as long as educated people are not embarrassed but rather proud of their poor levels of competence then this will not alter.

As Benn (1997 p.35) notes,

> 'It seems that despite calls for over 100 years for an approach to mathematics that interests and stimulates children at school, mathematics is still a subject that confuses, alienates and leads to failure.'

In looking at the development of numeracy skills there are many interesting theories of how and why people learn these basic concepts. Perhaps best known is the work of Piaget and Inhelder (Collis 1975) on how children learn which identified four stages of intellectual development, sensori-motor from 0-2 years of age, pre-operational from 2-6 years, concrete operational from 6-11 years and formal operations from 11. These they maintained corresponded to a child's grasp of concepts such as geometric shape and manipulation of fractions. They surmised that children were unable to grasp certain concepts until they had reached the appropriate stage of development.

I was undertaking further professional development at the time and one of my assignments focussed on educational theory so I chose to look more closely at the research of Piaget and Inhelder (Collis 1975). It was then that I started to think that there could be a correlation between their stages and the fact that some of my adult Numeracy students had never made much progress in educational terms beyond Primary level (Key Stage 2.) I found it very interesting to note how the formal key stages of education match closely to these four stages of intellectual development. I was also particularly interested in the fact that many of my adult Numeracy students talked about 'coping' with their own maths studies (or helping with children's homework) up until the start of Key Stage 3 but were unable to make much progress themselves into the Core Curriculum Level 1 and beyond.

I started to wonder if it was possible that they had not yet reached the higher levels of intellectual development that are maintained to occur from 11 to 14 years of age because of a lack of input of concrete experiences or that these students might need more in the way of the sort of active learning that goes on in primary education rather than the paper-based methods I had been using. At the time I was the Numeracy governor at my children's school and was invited to a briefing about the new National Numeracy Strategy (1999) which laid heavy emphasis on active learning methods. While I felt that many of the methods were not suitable for my adult learners I was interested in things like the use of multiplication and hundred squares and how they were being used to teach number relationships and started using those in my sessions.

Bruner's research (1964) into how people learn suggested three modes of representation: enactic, representing past events through a motor response; iconic, picturing an operation to recreate it mentally; and symbolic, writing maths equations. Bruner says these three stages are related to Piaget's stages of cognitive development and develop sequentially. He argued that you should present maths in this way when teaching, for instance when dealing with shape you should handle the shape, draw the shape and then write about it.

Ernest (1991) gives a different perspective on the failure of maths education arguing that the problem is one of philosophy. He argues that maths is not seen as relevant to people but as an abstract, absolute discipline which is not shaped by and cannot relate to everyday life and ordinary people. He argues for a change in philosophy for maths education away from the view that maths is an absolute truth not subject to change - which he calls an 'absolutist' view. In this view he argues (Ernest 2000,p.1),

"The outcome is therefore a philosophically sanctioned image of mathematics as rigid, fixed, logical, absolute, inhuman, cold, objective, pure, abstract, remote and ultra-rational. Is it a coincidence that this image coincides with the widespread public image of mathematics as difficult, cold, abstract, theoretical, ultra-rational, but important and largely masculine. Mathematics also has the image of being remote and inaccessible to all but a few super-intelligent beings with mathematical minds."

Benn (1997) reinforces this view. In her studies of the maths that people experienced in their everyday lives in craft activities like knitting she states, (Benn 1997 p.36),

> 'Many of these pleasurable processes are not thought of as mathematical. Indeed it sometimes seems that if people can do it, it is called common sense, if they can't, it is called mathematics.'

The alternative point of view is 'fallibilism', the view that emphasises the human side of mathematics. In this view mathematics is experienced as (Ernest 2000,p.1),

> 'warm, human, personal, intuitive, active collaborative, creative, investigational, cultural, historical, living, related to human situations, enjoyable, full of joy, wonder and beauty.'

This relates to the growing trend in maths education to teach children about the history of maths eg. the Egyptian number system, the work of early Greek mathematicians and to look at examples of 'mathematical art', to encourage them to see that maths has been created by humans and can be altered by humans.

I believe that adopting a multi-sensory, active learning approach to numeracy teaching can benefit all learners not just those with particular learning difficulties or disabilities. In particular the use of engagement through activities which involve concrete experiences. Much of maths has traditionally been taught in a didactic way involving explanations of concepts through board work then individual 'exercises' done by the students to demonstrate their understanding of the concept explained.

As a result of these thoughts I started to introduce further active and 'discovery learning' into my sessions using strategies such as these below.

- Handling of models of solid shapes when discussing properties of a shape
- The use of number lines for counting, multiplication tables and negative numbers
- Investigating the relationship between the circumference of circular objects and their diameter by using string and rulers to measure the dimensions
- Using multi - link cubes to investigate volume of different solid shapes
- Investigating the largest possible volume of cuboid that could be made from one sheet of paper.

Since coming to work at the University I have been privileged to work with many fantastically creative and inspiring teachers and trainee teachers and have gained many more ideas about activities for the classroom. As I no longer (sadly) teach adult Numeracy my role now is to pass on these ideas to future students and to continue to research and develop my own.

This has involved an ongoing search for ways to involve chocolate in my teaching not just for eating, it has to have a learning objective attached! So far I have found a use for Quality Street with Carroll and Venn diagrams for categorising shapes, Terry's chocolate orange for volume, radius and diameter work, bars of chocolate for fractions and the ever useful Smarties or Skittles for ratio, proportion, fractions and data handling work. I have also found many excellent activities through the Association of Teachers of Maths (details in references) I can highly recommend their Algebra jigsaws you will hear people getting excited about algebraic simplification!

As for the items mentioned in the title they are used as follows.

- The forty Ferrero Rocher are used in an activity I developed to look at the application of the formulae for the volume of sphere and cuboid to working out how much wasted space there is in two different sized boxes of the chocolates.

- The four kilograms of macaroni are for a brilliant activity that I found on the internet called 'The Cylinder Problem', (see references for details) which explores the relationship between basal area and height in the volume of cylinders. You can use rice for this one but pasta is easier to pick up off the floor!

- The Tower of Hanoi is one of a new set of wooden puzzles I have recently bought through a magazine subscription to *Classic Puzzles and Brainteasers* and is used to illustrate how algebraic solutions can be used for problem solving.

My quest for more ideas for active learning has renewed my enthusiasm for the subject engaging me in further professional study, this time in looking at metacognition in maths teaching and the history of the development of mathematical ideas. I and my students have also had far more fun in the last few years than anyone is probably supposed to in a maths classroom!

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